ECL 4340

POWER SYSTEMS

LECTURE 6

THREE-PHASE TRANSFORMERS AND MULTI-WINDING TRANSFORMERS

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ANNOUNCEMENT

- Read Chapter 3
- HW 3; due September 16 in Canvas.

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THREE-PHASE TRANSFORMER

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Three Phase Transformers

- There are 4 different ways to connect 3ϕ transformers: Y-Y, Δ-Δ, Y-Δ, Δ-Y
  - The reasons have to do with grounding and harmonics,
  - Only Y connections can be grounded
  - Mixing Y and Δ introduces a 30 degree phase shift
- Most high voltage generator step-up transformers (GSUs) are Δ on the generator side, grounded Y on the transmission side
- Most transmission to distribution is Δ on the transmission side, grounded Y on the distribution side

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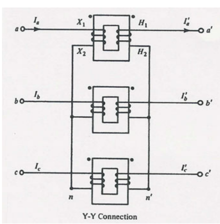
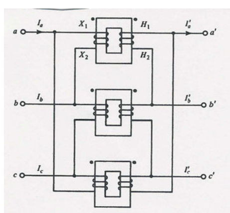
Three-Phase Transformer

There are 4 different ways to connect 3ϕ transformers

Y-Y



Δ-Δ



Usually 3ϕ transformers are constructed so all windings share a common core

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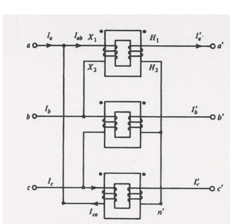
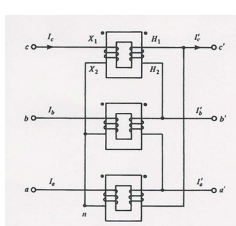
Three-Phase Transformer

Δ-Y



Y-Δ



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Three-Phase Transformer

American Standard (ANSI):

In either a Y-Δ or Δ-Y transformer, positive-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by 30°.

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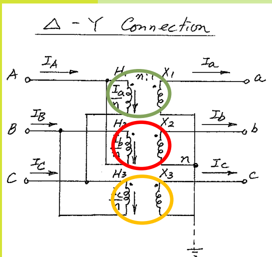
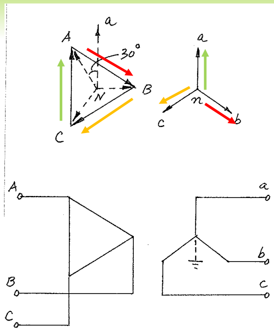
Three-Phase Transformer





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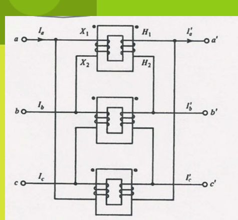
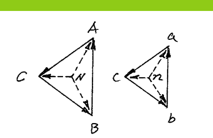
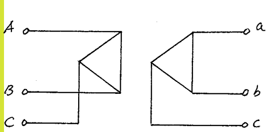
Three-Phase Transformer







The Δ-Δ connection has the advantage that one phase can be removed for repair or maintenance while the remaining phases continue to operate as a three-phase bank. This open Δ (or V) connection permits balanced three-phase operation with the kVA rating reduced to 58% of the original bank. (Why?)

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THREE-PHASE TRANSFORMER

Per-Unit Equivalent Circuits

By convention, we choose the base quantities as

1. A common S_{base} for both the H and X sides
2. The ratio of the voltage bases $V_{base\,H} / V_{base\,X}$ is selected to be equal to the ratio of the rated line-to-line voltages $V_{rated\,H_{LL}} / V_{rated\,X_{LL}}$

for all connections.

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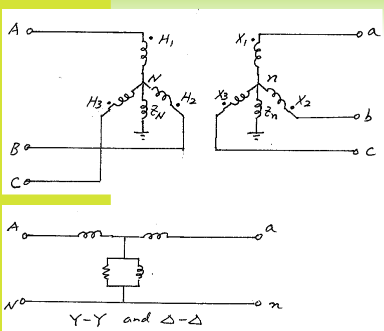
THREE-PHASE TRANSFORMER



For balanced 3φ neutral current is zero and there are no voltage drops across the neutral impedances. Therefore, the per-unit equivalent circuit of the Y-Y transformer is the same as the per-unit single-phase transformer.

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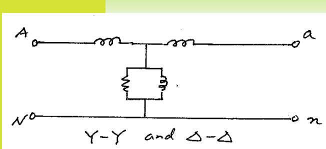
THREE-PHASE TRANSFORMER



- The per-unit equivalent circuit for Δ-Δ transformer is the same as that for Y-Y transformer. (Why?)

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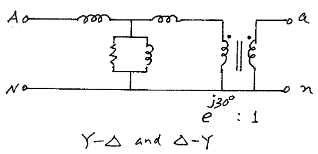
Three-Phase Transformer



For Δ–Y or Y–Δ transformer, high-voltage side leads the low-voltage side by 30° (ANSI Standard).

Therefore, we add an ideal transformer with a complex turns ratio of e^{j30°.

The per-unit impedances <u>do not</u> depend on the winding connections! (Why?)

Furthermore, the per-unit impedance of the 3ϕ transformer is <u>the same as</u> the per-unit impedance of the single-phase transformer! (Why?)

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Three-Phase Transformer

<u>Example</u>: Three single-phase transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance $X_{eq} = 0.10$ p.u., are connected to form a three-phase bank. Winding resistances and exciting currents are neglected. The high-voltage windings are connected in Y. A 3ϕ load connected to the high-voltage side draws 1000 MVA at 0.9 p.f. lagging, with $V_{AN} = 199.2\angle 0^\circ$ kV.

Determine the voltage V_{an} at the low-voltage bus if the low-voltage windings are connected (a) in Y, (b) in Δ.

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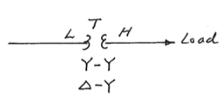
Three-Phase Transformer

Solution:



Use the transformer bank ratings as base quantities:

$$S_{base\,3\phi} = 3 \times 400 = 1200 \text{ MVA}$$

$$V_{base\,H_{LL}} = \sqrt{3}\,(199.2) = 345 \text{ kV}$$

$$I_{base\,H} = \frac{S_{base\,3\phi}}{\sqrt{3}\; V_{base\,H_{LL}}} = \frac{1200}{\sqrt{3} \times 345} = 2.008 \text{ kA}$$

Load current:

$$I_A = \frac{1000}{\sqrt{3} \cdot 345} \angle -\cos^{-1} 0.9 = 1.6735\angle -25.84^\circ \text{ kA}$$

$$= 0.8333\angle -25.84^\circ \text{ pu}$$

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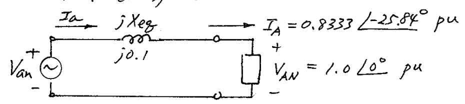
Three-Phase Transformer

(a) Y-Y transformer:



$$V_{an} = V_{AN} + (jX_{eq}) I_A = 1.0\angle 0^\circ + (j0.1)(0.8333\angle -25.84^\circ)$$

$$= 1.039\angle 4.14^\circ$$

$$\therefore V_{an} = 1.039\,(13.8) = \underline{14.34}\ kV$$

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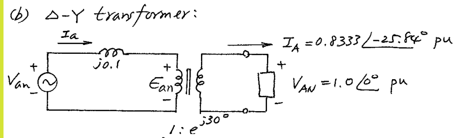
Three-Phase Transformer

(b) Δ-Y transformer:



$$E_{an} = e^{-j30^\circ} V_{AN} = 1.0\angle -30^\circ$$

$$I_a = e^{-j30^\circ} I_A = 0.8333\angle -25.84^\circ - 30^\circ$$

$$V_{an} = E_{an} + (jX_{eq}) I_a$$

$$= 1.0\angle -30^\circ + (j1.0)(0.8333\angle -25.84^\circ - 30^\circ)$$

$$= [\underbrace{1.0\angle 0^\circ + (j1.0)(0.8333\angle -25.84^\circ)}_{\text{same as in part (a)}}]\ \angle -30^\circ$$

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Three-Phase Transformer

$$V_{an} = E_{an} + (jX_{eq}) I_a$$

$$= 1.0\angle -30^\circ + (j1.0)(0.8333\angle -25.84^\circ - 30^\circ)$$

$$= [\underbrace{1.0\angle 0^\circ + (j1.0)(0.8333\angle -25.84^\circ)}_{\text{same as in part (a)}}]\ \angle -30^\circ$$

$$= 1.039\angle 4.14^\circ - 30^\circ$$

$$= 1.039\angle -25.86^\circ$$

$$\therefore V_{an} = 1.039\,(13.8/\sqrt{3}) = \underline{8.278}\ kV$$

NOTE:
- $|V_{an}|$ is the same in per-unit in Y-Y & Δ-Y.
- We often ignore the phase shift in calculating the magnitude of the voltage, $|V_{an}|$.

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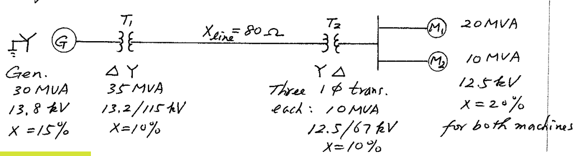
Three-Phase Transformer

Example:



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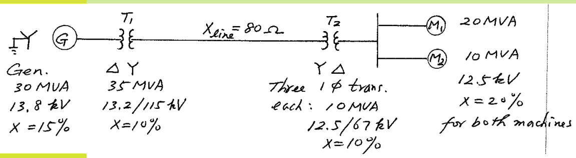
Three-Phase Transformer



Use the Generator ratings as the base quantities.

$S_{B\,3\phi} = 30$ MVA

Zone 1	Zone 2	Zone 3
$V_{B1} = 13.8$ kV	$V_{B2} = 13.8\left(\frac{115}{13.2}\right) = 120$ kV	$V_{B3} = 120\left(\frac{12.5}{\sqrt{3}\,(67)}\right) = 12.9$ kV
	$Z_{B2} = \frac{(120)^2}{30} = 480\ \Omega$	

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Three-Phase Transformer

Use the Generator ratings as the base quantities.

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	$Z_{B2} = \frac{(120)^2}{30} = 480\ \Omega$	

Line : $X_{Line} = \frac{80}{480} = 0.167$ pu

T_1 : $X_1 = (0.1)\left(\frac{13.2}{13.8}\right)^2\left(\frac{30}{35}\right) = 0.0784$ pu

T_2 : $X_2 = (0.1)\left(\frac{12.5}{12.9}\right)^2\left(\frac{30}{30}\right) = 0.0940$ pu

M_1 : $X_{M_1} = (0.2)\left(\frac{12.5}{12.9}\right)^2\left(\frac{30}{20}\right) = 0.282$ pu

M_2 : $X_{M_2} = (0.2)\left(\frac{12.5}{12.9}\right)^2\left(\frac{30}{10}\right) = 0.563$ pu

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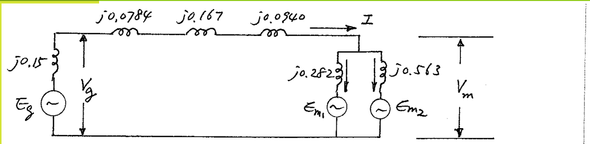
Three-Phase Transformer



Suppose the motors are operating with:
$M_1 = 16$ MW, $M_2 = 8$ MW @ 12.5 kV & 1.0 pf.
Then find the generator terminal voltage.

Since both machines are operating with the same powerfactor, total power is simply $P = 16 + 8 = 24$ MW at the unity power factor & at the common voltage of 12.5 kV.

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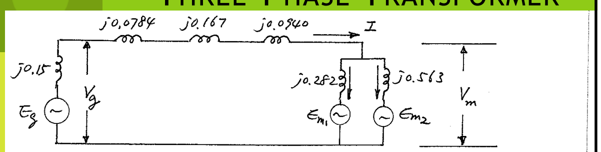
Three-Phase Transformer



In per-unit, the total power is

$$P_{pu} = \frac{24 \text{ MW}}{30 \text{ MVA}} = 0.8 \text{ p.u.}$$

The voltage at the motor bus:

$$V_{pu} = \frac{12.5}{12.9} = 0.969 \text{ p.u.}$$

The current drawn by the combined load:

$$I_{pu} = \frac{0.8}{0.969} = 0.826 \text{ p.u.}$$

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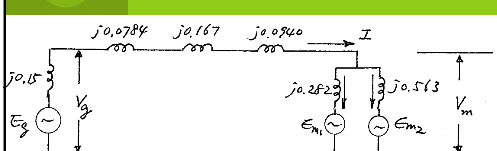
Three-Phase Transformer



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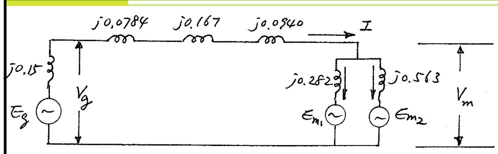
Three-Phase Transformer



Therefore, the generator terminal voltage is

$$V_g = V_m + I\,(j0.0784 + j0.167 + j0.0940)$$
$$= 0.969 + (0.826\angle 0^\circ)(j\,0.3394)$$
$$= 1.01\ \underline{\angle 16.1^\circ}\ \text{pu}$$

$$\therefore\ V_g = 1.01\,(13.8) = \underline{13.92}\ \text{kV}$$

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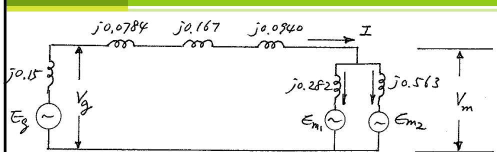
Three-Phase Transformer



<u>Remark</u>: If two machines are not operating at the same power factor, then current drawn by each machine can be computed first.

By adding the currents total load current can be obtained.

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Three Winding Transformers

- Many high voltage transformers have a third winding, called the tertiary winding; called three winding transformers
- There are a number of benefits in having 3 windings
  - Tertiary can be used to provide lower voltage electric service, including providing substation service for remote transmission substations; sometimes capacitors are connected to the tertiary
  - Helps with fault protection by reducing the zero sequence current providing higher zero sequence currents
  - When Δ-connected helps to reduce unbalanced and third harmonic issues

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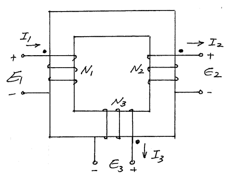
Multi-Winding Transformers

Multi-Winding Transformers



Generalizing the two-winding transformer, additional windings can be used to form multi-winding transformers.

For three-winding transformers the mmf balance in the common flux path is

$$N_1 I_1 = N_2 I_2 + N_3 I_3$$

or $$I_1 = \frac{N_2}{N_1} I_2 + \frac{N_3}{N_1} I_3$$

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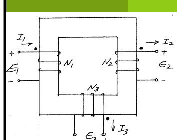
Multi-Winding Transformers



$$N_1 I_1 = N_2 I_2 + N_3 I_3$$

or $$I_1 = \frac{N_2}{N_1} I_2 + \frac{N_3}{N_1} I_3$$

In per-unit system this becomes

(1) $$I_{1pu} = I_{2pu} + I_{3pu}$$ (Why?)

For voltage, using the Faraday's law,

$$E_1 = j\omega N_1 \Phi, \quad E_2 = j\omega N_2 \Phi, \quad E_3 = j\omega N_3 \Phi$$

or $$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$$

Again, in per-unit system,

(2) $$E_{1pu} = E_{2pu} = E_{3pu}$$ (Why?)

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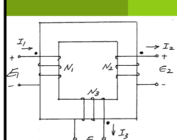
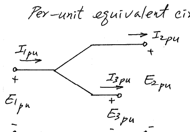
Multi-Winding Transformers



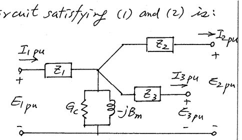
(1) $$I_{1pu} = I_{2pu} + I_{3pu}$$

(2) $$E_{1pu} = E_{2pu} = E_{3pu}$$

Per-unit equivalent circuit satisfying (1) and (2) is:



Ideal transformer



Practical transformer

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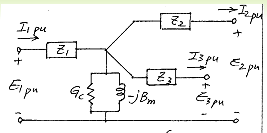
Multi-Winding Transformers



Practical transformer

The shunt admittance can be determined by the open-circuit test and the series leakage impedances can be found by the short-circuit test as was done in two-winding transformer.

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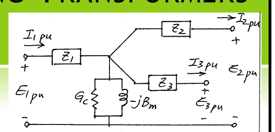
Multi-Winding Transformers



Practical transformer

Let Z_{ij} be the per-unit leakage impedance measured from winding i, with winding j shorted and winding k open, where $i, j, k = 1, 2, 3$.

Then

$$Z_{12} = Z_1 + Z_2$$
$$Z_{13} = Z_1 + Z_3$$
$$Z_{23} = Z_2 + Z_3$$

Solving these,

$$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$$
$$Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13})$$
$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$$

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Multi-Winding Transformers

Example 3.9: A 1φ tree-winding transformer:

N_1: 300 MVA, 13.8 kV
N_2: 300 MVA, 199.2 kV
N_3: 50 MVA, 19.92 kV

Leakage reactances, from short-circuit tests, are

$X_{12} = 0.1$ pu on 300 MVA, 13.8 kV base
$X_{13} = 0.16$ pu on 50 MVA, 13.8 kV base
$X_{23} = 0.14$ pu on 50 MVA, 199.2 kV base

Find the p.u. equivalent circuit using the base of 300 MVA and 13.8 kV on N_1.

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Multi-Winding Transformers

Leakage reactances, from short-circuit tests, are

$X_{12} = 0.1$ pu on 300 MVA, 13.8 kV base
$X_{13} = 0.16$ pu on 50 MVA, 13.8 kV base
$X_{23} = 0.14$ pu on 50 MVA, 199.2 kV base

Find the p.u. equivalent circuit using the base of 300 MVA and 13.8 kV on N_1.

Solution: System base: $S_B = 300$ MVA
Base voltages: $V_{B1} = 13.8$ kV, $V_{B2} = 199.2$ kV, $V_{B3} = 19.92$ kV

New p.u. values:
$X_{12} = 0.1$ pu
$X_{13} = (0.16)\left(\frac{300}{50}\right) = 0.96$ pu
$X_{23} = (0.14)\left(\frac{300}{50}\right) = 0.84$ pu

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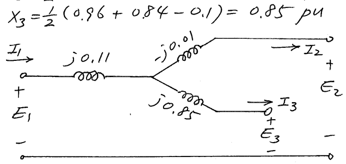
Multi-Winding Transformers

Reactances are
$X_1 = \frac{1}{2}(0.1 + 0.96 - 0.84) = 0.11$ pu
$X_2 = \frac{1}{2}(0.1 + 0.84 - 0.96) = -0.01$ pu
$X_3 = \frac{1}{2}(0.96 + 0.84 - 0.1) = 0.85$ pu



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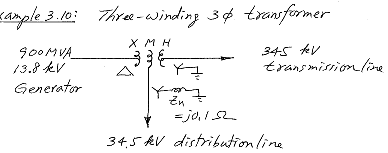
Multi-Winding Transformers

Example 3.10: Three-winding 3φ transformer



Draw the per-unit circuit using the 3φ base of generator as the system base.

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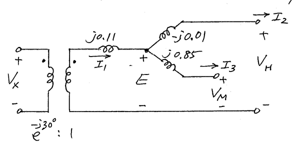
Multi-Winding Transformers

Solution: In American Standard M and H (in Y) should lead X (in Δ) by 30°.



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Multi-Winding Transformers

Example: Transmission line delivers 450 MVA at 345 kV with 0.8 pf lagging.
Resistive load of 150 MW at 34.5 kV in the distribution line.
Find the voltage at the generator terminal.

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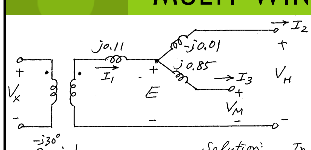
Multi-Winding Transformers



Solution: In per-unit, the transmission line current:

$$I_2 = \frac{450}{900}(0.8 - j0.6) = 0.4 - j0.3 = 0.5\angle{-36.87°}\ pu$$

Resistance in distribution line is

$$R = (1.0)\left(\frac{900}{150}\right) = 6.0\ pu$$

Voltage at E due to the load current I_2:

$$E = V_H + Z_2 I_2 = 1.0 + (-j0.01)(0.4 - j0.3)$$
$$= 1.0 - 0.003 - j0.004 = 0.997 - j0.004$$
$$\simeq 0.997\angle{-0.23°}$$

Current in the distribution line:

$$I_3 = \frac{E}{6 + j0.85} = \frac{0.997\angle{-0.23°}}{6.06\angle{8.06°}} = 0.165\angle{-8.29°}$$

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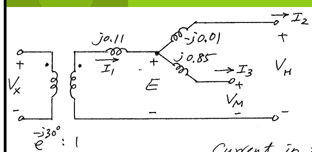
MULTI-WINDING TRANSFORMERS



Current in the low voltage winding:

$$I_1 = I_2 + I_3 = 0.4 - j0.3 + (0.163 - j0.024)$$
$$= 0.563 - j0.324 = 0.65\angle -29.92°$$

Voltage at the generator terminal:

$$V_x = (e^{-j30°})(E + Z_1 I_1)$$
$$= (e^{-j30°})(0.997 - j0.004 + (j0.11)(0.563 - j0.324))$$
$$= (e^{-j30°})(\quad '' \quad + j0.062 + 0.0356)$$
$$= (e^{-j30°})(1.0326 + j0.058) = 1.0342\angle 3.215° - 30°$$
$$= 1.0342\angle -26.79°$$

$$|V_x| = (1.0342)(13.8\ kV) = \underline{\underline{14.27}}\ kV$$

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